

تابع اولیه انتگرال های مختلفی را در جدول ذیل ذکر نمودیم. برای انتگرال گیری از تابع دیگر می بایست انتگراله را به یکی از اشکال زیر درآورد.

$$\int dx = x + C \quad (1.1)$$

$$\int a dx = ax + C \quad (\text{عدد ثابت } a) \quad (2.1)$$

$$\int x^r dx = \frac{x^{r+1}}{r+1} + C \quad (\text{عدد حقیقی } r \neq -1) \quad (3.1)$$

$$\int \frac{1}{x} dx = \ln|x| + C \quad (x \neq 0) \quad (4.1)$$

$$\int \frac{1}{x-a} dx = \ln|x-a| + C \quad (x \neq a) \quad (5.1)$$

$$\int e^x dx = e^x + C \quad (6.1)$$

$$\int a^x dx = \frac{a^x}{\ln a} + C \quad (a > 0, a \neq 1) \quad (7.1)$$

$$\int \frac{1}{\sqrt{a^x - x^x}} dx = \arcsin\left(\frac{x}{a}\right) + C \quad (a > 0) \quad (8.1)$$

$$\int \frac{1}{\sqrt{a^x + x^x}} dx = \sinh^{-1}\left(\frac{x}{a}\right) + C \quad (a > 0) \quad (9.1)$$

$$\int \frac{1}{a^x + x^x} dx = \frac{1}{a} \arctan\left(\frac{x}{a}\right) + C \quad (a \neq 0) \quad (10.1)$$

$$\int \frac{1}{a^x - x^x} dx = \frac{1}{\ln a} \ln \left| \frac{a+x}{a-x} \right| + C \quad (a \neq 0) \quad (11.1)$$

$$\int \frac{1}{(x+a)(x+b)} dx = -\frac{1}{a-b} \ln \left| \frac{x+a}{x+b} \right| + C \quad (a \neq b) \quad (12.1)$$

ضروری است که ذکر کنیم برای هر تابع پیوسته f همواره داریم:

$$\int df = f$$

مثال ۱۰.۱۱. حاصل انتگرال های زیر با استفاده از جدول فوق بدست آمده است.

$$\begin{aligned}
 (a) \quad & \int x^{\frac{1}{2}} + 6x^{\frac{1}{3}} + x - 9 \, dx = \frac{x^{\frac{5}{2}}}{5} + 6 \frac{x^{\frac{4}{3}}}{3} + \frac{x^2}{2} - 9x + C \\
 (b) \quad & \int \sqrt{v}x^{\frac{1}{2}} + x^{\frac{1}{3}} + \frac{1}{x} - \frac{2}{x^{\frac{1}{2}}} \, dx = \int \sqrt{v}x^{\frac{1}{2}} + x^{\frac{1}{3}} + \frac{1}{x} - 2x^{-\frac{1}{2}} \, dx \\
 & = \sqrt{v}x^{\frac{1}{2}} + \frac{x^{\frac{4}{3}}}{3} + \ln|x| - 2 \frac{x^{-\frac{1}{2}}}{-\frac{1}{2}} + C \\
 (c) \quad & \int 4x^{\frac{5}{2}} + 3\sqrt[3]{x^{\frac{1}{2}}} - \frac{6}{\sqrt[3]{x^{\frac{1}{2}}}} \, dx = \int 4x^{\frac{5}{2}} + 3x^{\frac{1}{6}} - 6x^{\frac{-1}{6}} \, dx \\
 & = 4 \frac{x^{\frac{7}{2}}}{7} + 3 \frac{x^{\frac{7}{6}}}{\frac{7}{6}+1} - 6 \frac{x^{\frac{-5}{6}}}{\frac{-5}{6}+1} + C \\
 & = \frac{8}{7}x^{\frac{7}{2}} + \frac{18}{7}\sqrt[3]{x^{\frac{1}{2}}} - \frac{6}{5}\sqrt[6]{x^5} + C \\
 (d) \quad & \int \frac{\sqrt{x}}{x} - v^{x+1} + \frac{1}{x} \, dx = \int v^{x-\frac{1}{2}} - 2 \times v^x + \frac{1}{x} \, dx \\
 & = \frac{1}{2}\sqrt{x} - \frac{v^{x+1}}{\ln v} + 2\ln|x| + C \\
 (e) \quad & \int \frac{4x^{\frac{1}{2}} - 9x}{x^{\frac{1}{2}}} \, dx = \int \frac{4x^{\frac{1}{2}}}{x^{\frac{1}{2}}} - \frac{9x}{x^{\frac{1}{2}}} \, dx \\
 & = \int 4x - 9 \frac{1}{x} \, dx \\
 & = 4x^2 - 9 \ln|x| + C \\
 (f) \quad & \int \frac{dx}{\sqrt{9-x^2}} = \frac{1}{\lambda} \ln \left| \frac{\sqrt{9+x}}{\sqrt{9-x}} \right| + C \\
 (g) \quad & \int \frac{dx}{9+x^2} = \frac{1}{\sqrt{9}} \arctan\left(\frac{x}{\sqrt{9}}\right) + C \\
 (h) \quad & \int \frac{dx}{9-x^2} = \frac{1}{\sqrt{9}} \ln \left| \frac{\sqrt{9+x}}{\sqrt{9-x}} \right| + C \\
 (i) \quad & \int v^x + v^{-x} \, dx = \frac{v^x}{\ln v} + \frac{\left(\frac{1}{v}\right)^x}{\ln \frac{1}{v}} + C \\
 (j) \quad & \int \frac{x^{\frac{1}{2}} + x^{\frac{1}{3}} + 1}{x^{\frac{1}{2}}} \, dx = \int x + \frac{1}{x} + x^{-\frac{1}{2}} \, dx \\
 & = \frac{x^2}{2} + \ln|x| + \frac{x^{-\frac{1}{2}}}{-\frac{1}{2}} + C \\
 (k) \quad & \int \frac{1}{x^{\frac{1}{2}} - 5x + 6} \, dx = \int \frac{1}{(x-2)(x-3)} \, dx \\
 & = -\frac{1}{-2+3} \ln \left| \frac{x-2}{x-3} \right| + C \\
 & = \ln \left| \frac{x-2}{x-3} \right| + C
 \end{aligned}$$

$$(a) \int x^{\delta} + \varepsilon x^{\gamma} + x - \alpha \, dx = \frac{x^{\delta}}{\delta} + \varepsilon \frac{x^{\gamma}}{\gamma} + \frac{x}{2} - \alpha x + C$$

$$(b) \int \sqrt[\gamma]{x^{\delta}} + x^{\gamma} + \frac{1}{x} - \frac{\gamma}{x^{\gamma}} \, dx = \int \sqrt[\gamma]{x^{\delta}} + x^{\gamma} + \frac{1}{x} - 2x^{-\gamma} \, dx \\ = \sqrt[\gamma]{x^{\delta}} + \frac{x^{\gamma}}{\gamma} + \ln|x| - 2 \frac{x^{-\gamma}}{-\gamma} + C$$

$$(c) \int \sqrt[\gamma]{x^{\delta}} + \sqrt[\gamma]{x^{\gamma}} - \frac{\delta}{\sqrt[\gamma]{x^{\gamma}}} \, dx = \int \sqrt[\gamma]{x^{\delta}} + \sqrt[\gamma]{x^{\gamma}} - \delta x^{\frac{-\gamma}{\gamma}} \, dx \\ = \frac{\delta}{\gamma} x^{\frac{\delta}{\gamma}} + \frac{\gamma}{\gamma+1} x^{\frac{\gamma+1}{\gamma}} - \delta \frac{x^{\frac{-\gamma}{\gamma}+1}}{\frac{-\gamma}{\gamma}+1} + C \\ = \frac{\gamma}{\gamma} x^{\delta} + \frac{1}{\gamma} \sqrt[\gamma]{x^{\gamma}} - \frac{\delta}{\gamma} \sqrt[\gamma]{x^{\delta}} + C$$

$$(d) \int \frac{\sqrt[\gamma]{x}}{x} - \sqrt[\gamma]{x+1} + \frac{\gamma}{x} \, dx = \int \sqrt[\gamma]{x^{\frac{-1}{\gamma}}} - 2 \times \sqrt[\gamma]{x} + \frac{1}{x} \, dx \\ = \lambda \sqrt[\gamma]{x} - \frac{\sqrt[\gamma]{x+1}}{\ln \gamma} + \gamma \ln|x| + C$$

$$(e) \int \frac{\sqrt[\gamma]{x^{\gamma}-\alpha x}}{x^{\gamma}} \, dx = \int \frac{\sqrt[\gamma]{x^{\gamma}}}{x^{\gamma}} - \frac{\alpha x}{x^{\gamma}} \, dx \\ = \int \sqrt[\gamma]{x} - \alpha \frac{1}{x} \, dx \\ = \gamma x^{\frac{1}{\gamma}} - \alpha \ln|x| + C$$

$$(f) \int \frac{dx}{\sqrt[\gamma]{x-x^{\gamma}}} = \frac{1}{\lambda} \ln \left| \frac{\sqrt[\gamma]{x}+x^{\frac{\gamma}{\gamma-1}}}{\sqrt[\gamma]{x}-x^{\frac{\gamma}{\gamma-1}}} \right| + C$$

$$(g) \int \frac{dx}{\sqrt[\gamma]{1+x^{\gamma}}} = \frac{1}{\gamma} \arctan \left(\frac{x}{\sqrt[\gamma]{\gamma}} \right) + C$$

$$(h) \int \frac{dx}{\sqrt[\gamma]{1-x^{\gamma}}} = \frac{1}{\gamma} \ln \left| \frac{\sqrt[\gamma]{\gamma}+x^{\frac{\gamma}{\gamma-1}}}{\sqrt[\gamma]{\gamma}-x^{\frac{\gamma}{\gamma-1}}} \right| + C$$

$$(i) \quad \int \gamma^x + \gamma^{-x} dx = \frac{\gamma^x}{\ln \gamma} + \frac{(\frac{1}{\gamma})^x}{\ln \frac{1}{\gamma}} + C$$

$$(j) \quad \begin{aligned} \int \frac{x^r + x^{-r} + 1}{x^r} dx &= \int x + \frac{1}{x} + x^{-r} dx \\ &= \frac{x^r}{r} + \ln|x| + \frac{x^{-r}}{-r} + C \end{aligned}$$

$$(k) \quad \begin{aligned} \int \frac{1}{x^r - \alpha x + \gamma} dx &= \int \frac{1}{(x - \gamma)(x - \gamma)} dx \\ &= -\frac{1}{-\gamma + r} \ln \left| \frac{x - \gamma}{x - \gamma} \right| + C \\ &= \ln \left| \frac{x - \gamma}{x - \gamma} \right| + C \end{aligned}$$

تمرین ۲.۱. حاصل انتگرال‌های زیر را بیابید.

- (a) $\int x^r + \alpha x^r - rx dx$, (b) $\int \frac{1}{x^a} - \alpha x^r + rx^r + \frac{r}{x} dx$
- (c) $\int \frac{x^y - \alpha x^r + rx^r + \alpha}{x^r} dx$, (d) $\int x(x^r + x^{-r} - \alpha) dx$
- (e) $\int \frac{dx}{\sqrt[4]{x-x^r}}$, (f) $\int \sqrt[4]{x} - \alpha^x + \frac{r}{x^r} dx$
- (g) $\int (x^r + \alpha)(rx^r - r) dx$, (h) $\int \frac{1}{x^r - x - \gamma^2} dx$
- (i) $\int r^x + \alpha^{x+r} - e^x dx$, (j) $\int \frac{r}{\alpha^x - x^r} dx$
- (k) $\int \frac{x^a - rx^r + x^r + rx^r}{x^r} dx$, (l) $\int \frac{r}{x^r - rx + \gamma^2} dx$
- (m) $\int \frac{\sqrt{x} + \sqrt[4]{x^r} + rx^r - rx}{\sqrt[4]{x}} dx$, (n) $\int \frac{rx + r}{x^r - rx + \gamma} dx$